MAN/HEP/2012/016

Multiple dark matter scenarios from ubiquitous stringy throats

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We discuss the possibility of having multiple Kaluza-Klein (KK) dark matter candidates which arise naturally in generic Type-IIB string theory compactification scenarios. These dark matter candidates reside in various throats of the Calabi-Yau manifold. In principle, they can come with varied range of masses in four-dimensions depending upon the hierarchical warping of the throats. We show that consistency with cosmological bounds and four-dimensional effective theory description imposes strong constraints on the parameter space and the geometry of the throats. With a rather model-independent approach, we find that the mass scales allowed for the KK dark matter particles in various throats can vary between 0.1 eV and 10 TeV. Thus, there could be simultaneously more than one kind of cold (and possibly warm and hot) dark matter components residing in the Universe. This multiple dark matter scenario could weaken the bound on a conventional supersymmetric dark matter candidate and even act as an extra relativistic degree of freedom.

I. INTRODUCTION

Several astrophysical and cosmological observations have established the existence of Dark Matter (DM) in our Universe [1], and hence, a possible hint for New Physics beyond the Standard Model (SM). Although all the evidence so far is based only on its gravitational interaction, the combined data from cosmological and particle physics sources require any standard DM candidate to satisfy the following conditions: (i) it must be stable on cosmological time scales and have the right pressure (to help large scale structure formation), (ii) it must have the right relic density (not to overclose the Universe), and (iii) it must be very weakly interacting with the SM particles (to satisfy the direct and indirect detection constraints).

There exist a plethora of particle DM candidates [2] with masses ranging from 10^{-5} eV to as high as the Grand Unified Theory (GUT) scale $\sim 10^{16}$ GeV, motivated by various New Physics scenarios. The most studied DM candidates are the Weakly Interacting Massive Particles (WIMPs) in the mass range $\sim$ 1 GeV - 10 TeV, as they occur almost naturally in very well-motivated particle physics theories, and have rich phenomenological implications. For example, one of the most popular WIMP candidates is the lightest supersymmetric particle (LSP) in various supersymmetric models with conserved R-parity [3] which were originally proposed to provide an elegant solution to the gauge hierarchy problem of the SM.

Various phenomenological extra-dimensional models provide an alternative potential solution to the gauge hierarchy problem, and also a viable WIMP DM candidate in the form of the lightest Kaluza-Klein (KK) mode. In the simplest Universal Extra Dimensional (UED) models, the lightest KK particle (LKP) remains stable, and hence a possible DM candidate [4], due to KK-parity, which is a remnant of the translational invariance in the extra dimension, after orbifolding (i.e., a Z_2 symmetry) is imposed to obtain chirality. In the more realistic Randall-Sundrum (RS) model [5], where the hierarchy between the electroweak and Planck scales arises naturally from a warped geometry of the extra-dimensional space, the stability of the KK modes can be ensured in a bottom-up approach by imposing a gauged symmetry [6], or in a top-down approach by assuming the (approximate) preservation of isometries [7].

On the other hand, it is desirable to study a model of DM where the DM candidates arise naturally from more complete and fundamental theories capable of also describing, in a coherent and unified framework, the other stages of the evolution of our Universe, such as inflation [8] (for a review, see e.g., [9, 10]). Arguably, the superstring theory is the most developed Planck scale theory that can make phenomenological connection at low scales. It provides a viable description of our four-dimensional spacetime by compactifying six extra dimensions on a Calabi-Yau manifold.

In the so-called *flux compactification* models, the internal Calabi-Yau manifold presents background fluxes arising from three-forms piercing through the internal cycles, and numerous sources of energy such as orientifold planes, D-branes, $\bar{D}$-branes, NS-branes, etc. (for a review, see e.g., [11]). The fluxes, branes and orientifold planes typically reduce the $\mathcal{N}=2$ supersymmetry preserved by the Calabi-Yau compactification to $\mathcal{N}=1$ or null supersymmetry (SUSY). The backreaction from the fluxes transform the metric from a perfectly factorized one into a warped geometry. Strongly warped regions develop generically when the fluxes are supported on cycles localized in small

regions of the internal manifold. A typical example is the case of cycles that can shrink to a conifold singularity (which can then be deformed), arguably the most generic singularity in Calabi-Yau spaces, which can occur in large numbers due to the presence of many three-cycles.

It is now a prediction of string theory that flux configurations and the generic presence of several conifold points lead to a multiple throat scenario [12, 13]. Such a setup is also phenomenologically preferred, as throats with different warpings give rise to different hierarchies, thus solving the hierarchy puzzle in particle physics as well as providing viable inflationary models in the hidden sectors or in the bulk [10, 14, 15]. However, the string models generically predict many hidden sectors which makes it difficult to obtain the branching ratios among visible and hidden sectors after preheating and reheating.

This paper is motivated from the observation that as a natural consequence of the multi-throat scenario, ubiquitous in generic string compactifications, every hidden throat (i.e., different from the throat where the SM resides) can harbor stable matter in the form of KK modes [1], whose mass range will depend on how deep the throats are (gravitational redshifting) [15]. This in addition to the usual visible sector DM candidate in the (MS)SM throat, leads naturally to a multiple DM scenario [2]. The stability of the KK modes against decaying into SM fields will be ensured by the suppression of decay rates due to the necessary tunneling between throats in a warped geometry [3].

In this paper, we present a calculation of the resulting relic density for these KK modes populating the various available throats, and point out important constraints on the underlying string theory models. We emphasize that our approach is independent of the details of the string inflation models and the dynamics of reheating or preheating. We derive stringent bounds on the parameter space and the throat geometry even when the branching ratio of the inflaton to visible and hidden sector degrees of matter cannot be determined so easily from the string top-down approach.

We however find that there are allowed regions in parameter space for this multiple DM scenario to work and we obtain the possible range of masses for the DM candidates and the bounds on the local string scales. We also discuss the bounds on the inflationary scale for certain classes of models. Finally, we point out some interesting phenomenological consequences of this multiple DM scenario.

The paper is organized as follows: in Section II, we discuss the stability of the DM candidates. In Section III, we calculate their relic density and obtain the bounds on the parameter space for our multiple DM scenario. In section IV, we discuss some interesting phenomenological implications. Our conclusions are given in Section V.

II. STABILITY OF THE KK MODES IN MULTIPLE THROATS

In the multi-throat scenario we are entertaining, the new DM candidates are the KK modes residing in throats *different* from the SM one [4]. The stability against decaying to and interacting with the SM particles will be ensured by the smallness of the couplings to the SM fields, and hence of the decay rates, as they are suppressed by the necessity of quantum-mechanical tunneling among the separate throats. Here we summarize the main features of such tunneling.

The local metric for each throat far from the tip can be written as a warped product with the generic form:

$$ds^2 = H(r)^{-1/2}(g_{\mu\nu}dx^\mu dx^\nu) + H(r)^{1/2}(dr^2 + r^2 ds^2_{X_5}), \tag{1}$$

where $\mu, \nu = 0, 1, 2, 3$ run through the 4-dimensional metric. The radial coordinate r reaches a minimum $r_{\min}$ at the tip of the throat, and thus the local string scale at the tip of the i-th throat is

$$M_i \equiv M_s H(r_{\min})^{-1/4}\Big|_{i\text{-th throat}} \equiv M_s h_i, \tag{2}$$

where h_i is the maximum warping factor of the i-th throat ($h_i = 1$ corresponds to no warping), and $M_s = \ell_s^{-1}$ is the ten-dimensional string mass scale, which is usually taken to be slightly smaller than the reduced Planck mass: $M_{\rm PL} = \ell_{\rm PL}^{-1} = 2.4 \times 10^{18}$ GeV.

The lightest KK mass will be given by [13]

$$m_{\mathrm{KK}(i)} \sim \frac{h_i}{R_i} \tag{3}$$

where R_i is the curvature radius of the i-th throat. The spacing between the KK masses is also given by

[1] We assume for simplicity no extra branes, and hence, no other matter content in the hidden throats although any form of matter in these throats could in principle represent a DM candidate.

[2] For an incomplete list of other multiple DM scenarios in various particle physics models, see e.g. [16, 17].

[3] Note that this is different from the warped scenarios in [7, 13], where the LKP resides only *within* the SM throat and its stability is guaranteed by the (approximate) preservation of isometries of the throat.

[4] We will comment on the possibility of having long-lived KK modes also in the SM throat in section III.

$\frac{h_i}{R_i}$. Note that with multiple throats, we will have multiple KK dark matter candidates. From the 4-dimensional point of view the largest mass which we can imagine will be always bounded by the compactification scale, while the lowest KK mass can be very small (see e.g. [18]) and is limited only by the consistency of the model. From the top-down approach, given a number C of conifold points in the Calabi-Yau manifold, the average number of throats with a warping $h < \bar{h}$ is [12]

$$\bar{N}(h < \bar{h}|C) \simeq \frac{C}{3\log(1/\bar{h})}. \tag{4}$$

Quantum mechanical tunneling will occur among the separate throats as long as the quantum numbers of the tunneling modes match in the two throats. The decay rate depends on the equation of motion of the modes, and thus on the specific form of the metric (1) generating the gravitational potential wall between the throats. The prototypical throat metric is the Klebanov-Strassler solution [19] for which, however, the exact solution of the mode equation is not known. Useful approximations have been used in the literature in order to obtain the decay rate, as discussed below.

By approximating the throat as $AdS_5 \times X_5$, and the bulk region connecting two throats as a "Planck brane" akin to the case of the RS model [5], the tunneling rate from the i-th throat to any other neighboring throat is given by [13]

$$\Gamma^{(RS)}_{\mathrm{tunn}(i)} = (m_{\mathrm{KK}(i)} R_i)^4 \frac{r_{\mathrm{min}_{(i)}}}{R_i^2} \sim \frac{h_i^5}{R_i}. \tag{5}$$

Other modelings of the compactified geometry with different form of the warping factor $H(r)$ in Eq. (1) or different modeling of the bulk region between throats lead to even more suppressed tunneling rates. For example, [15] presents two possibilities giving rise to the decay rates

$$\Gamma^{(\mathrm{I})}_{\mathrm{tunn}(i)} \sim h_i^9 R_i^{-1} \quad \text{and} \quad \Gamma^{(\mathrm{II})}_{\mathrm{tunn}(i)} \sim h_i^{17} R_i^{-1}. \tag{6}$$

We learn from Eq. (5) that tunneling from a longer throat (small warping h_i) to a shorter one is highly disfavored. For what concerns the DM properties, this means that the lowest KK mode in the i-th throat will be stable against interacting with other throats provided the throat is long enough to ensure that the tunneling life-time is larger than the age of the Universe, i.e. $\Gamma_{\mathrm{tunn}(i)} < H_0 = 100h$ km.s^{-1}.Mpc^{-1} (with $h = 0.72 \pm 0.03$ [20]).

The KK modes in the throats could still be unstable under decay into massless bulk modes, i.e. gravitons, thus overproducing such massless radiation. However, this does not happen because of two reasons [14, 15]: first, the annihilation cross-section of the KK modes to gravitons is Planck-suppressed as $\sigma_g \sim M_{\mathrm{PL}}^{-2}$ (whereas the KK self-interactions are warp-enhanced, and thus essentially suppressed by the local string scale), and further reduced by the fact that only modes carrying oppositely conserved internal momentum can annihilate. Second, gravitons are rapidly redshifting ($\sim a^{-4}$) compared with the massive KK modes ($\sim a^{-3}$), when the latter become non-relativistic.

III. RELIC DENSITY OF THE KK DARK MATTER

The production of the KK modes localized in the throats depends on the stringy realization of inflation and (p)reheating. However, the post-production evolution of the KK modes in the hidden throats shows remarkable model-independent features, because of the particular background geometry leading to the suppression of interaction among different throats due to tunneling.

Indeed, this scenario has a striking difference compared to the standard WIMP DM picture. In the latter case, the DM candidate participates in the *same* thermal history as the SM degrees of freedom and evolve independently only after freeze-out (which determines their relic density). Instead, in the multi-throat scenario, the DM particles in different long enough throats are decoupled from each other and from the SM degrees of freedom by tunneling suppression of the interaction cross-sections between them [5]. As shown later, their relic density is determined essentially by the initial conditions[6].

Such dependence on the initial conditions of production can be efficiently parametrized by subdividing the models into two general classes on the basis of the mechanism of reheating and production of the KK modes. Therefore, our analysis can be rather model-independent.

In one class, the energy which excites the KK modes in the hidden throats and reheats the SM in its own throat is initially localized essentially in a single throat, from which it is transferred to the others by *tunneling*. A typical example is the D-$\bar{D}$ inflation

[5] Gravitational interactions, though not tunneling suppressed, are suppressed by M_{PL}^2 and hence unable to lead to thermal equilibrium among the throats.

[6] In the multi-throat scenario, the DM in each throat has its own thermal history, and in case it reaches thermal equilibrium in the throat, its own temperature. These possible temperature differences might affect the counting of the total relativistic degrees of freedom g_* and g_{*S} entering respectively the formulas for the energy density and entropy. However, the dependence of the relic density on these quantities will turn out to be weak.

models with the reheating-through-tunneling mechanism studied in [14, 15]. Note however that our analysis is independent of the specific models: it applies to all models where the energy responsible for reheating of the DM and SM fields resides initially all in one throat, independently of how and where exactly inflation occurred. We will call this throat the "initial throat" (indicated with a label "in"), and these class of models "throat-reheating" [7].

The other class of models (which we call "bulk-reheating") instead includes models where the energy responsible for reheating does not reside in one specific initial throat, but is spread over the internal manifold. Typical examples are those where the reheating field is a bulk inflaton or a bulk modulus, which decay releasing its energy all over the warped Calabi-Yau manifold. These models, however, must reheat the SM efficiently, therefore requiring different branching ratios among throats, and among bulk and throats. Note that our analysis will be independent of specific models also in this case. It may even be the case that inflation has occurred in one specific throat, but couples more efficiently to bulk moduli or fields such as gravitinos, which then, through their decay, will reheat the other degrees of freedom from the bulk.

As our analysis will be independent of the specific features of perturbative reheating or preheating, we use the term "reheating" in a generic way, encompassing all possible ways that produce the KK modes within the hidden throats.

Summarizing, the essential features of the models relevant for our analysis are: 1) long enough throats are decoupled from each other because of tunneling suppression, and 2) models can be divided in two broad classes depending on if 2A) the energy responsible for reheating is initially localized in one specific throat from where it has to tunnel out, or if 2B) the energy responsible for reheating can be released through direct couplings and does not require tunneling to excite the other degrees of freedom.

A. Reheating in one throat

In this scenario the energy density responsible for reheating is initially localized in one specific throat from where it has to tunnel out. The typical example is the scenario of D-$\bar{D}$ inflation in one throat. In that case, the end of inflation is brought about by the annihilation of the D and $\bar{D}$ branes. Their decay products are massive fundamental closed strings, which decay until KK modes that quickly thermalize are left over [14, 15]. Then these modes tunnel to the other throats reheating them [8].

The largest part of the energy will be tunneled into the longest throat since it has the densest spectrum, which favors tunneling as the level matching between masses is easier. The remaining degrees of freedom end up in sufficiently long throats since the tunneling out of them is very suppressed. The shallow throats will be practically empty, because they receive less tunneled energy from the initial throat to start with, and the degrees of freedom that might be excited will anyway tunnel out from them in a short time-scale.

Note that since the initial throat has to be shallower than the other throats for tunneling to occur, even the lightest modes in the initial throat will tunnel to modes which are not the lightest ones in their respective throats because of the level-matching. These higher modes in the final throats are in general unstable and decay into the lighter ones much more rapidly than the Hubble rate after tunneling from the initial (in) throat provided that

$$h_{\rm in}^3 > R_{\rm in}^2 M_i^2 \sqrt{R_{\rm in}^2 M_{\rm PL} \Gamma_{\rm in,\ tunn}} \tag{7}$$

which, as we will see later, is easily satisfied (see Fig. 3). Hence, we can safely assume that the long hidden throats harbor only DM candidates in the form of the lightest KK modes.

Assuming that the SM radiation dominates soon after tunneling, and is in thermal equilibrium (the necessary conditions will be discussed later), and recalling that the different throats are decoupled from each other by tunneling suppression, we can derive an equation for the relic density of the KK dark matter candidates:

$$\begin{aligned}\Omega_X h^2 &= \frac{1.3\times 10^{-4}}{g'_*\, g_*(T_{\rm in,\ tunn})} \frac{\sum_i m_{X_i} n_{X_i}(t_{\rm in,\ tunn})}{T^3_{\rm in,\ tunn} T_0} \\ &= \frac{1.8\times 10^{8}}{g'_*}\left(\frac{T_{\rm in,tunn}}{1\ {\rm GeV}}\right) F_X\ , \end{aligned} \tag{8}$$

where T_0 is today's temperature, $t_{\rm in,tunn}/T_{\rm in,tunn}$ is the time/temperature after tunneling from the initial throat, $g'_* \equiv \frac{g_*(T_0) g_{*S}(T_{\rm in,\ tunn})}{g_{*S}(T_0) g_*(T_{\rm in,\ tunn})}$ (with g_*, g_{*S} the total relativistic degrees of freedom entering respectively

[7] We will not consider the case when the SM and the initial throat coincide, as the analysis in this case is similar to that in the RS models [6, 7].

[8] The modes at tunneling are non-relativistic. Indeed, if we assumed that they were relativistic, the corresponding temperature $T_{\rm in,tunn}$ would be found to be smaller than the mass of even the lightest KK mode ($m_{KK({\rm in})}$). For example, even for the less suppressed decay rate in the case of RS-like throat geometry, the ratio is $\frac{T_{\rm in,tunn}}{m_{KK({\rm in})}} = \sqrt{R_{\rm in} M_{\rm PL}}\, h_{\rm in}^{3/2}$, which is generally much smaller than one.

the relation for energy density and entropy), $n_i(t)$ is the number density of the DM particles X_i, and

$$F_X \equiv \sum_i \left(\frac{m_{X_i}}{m_{KK(\mathrm{in})}}\right)\left(\frac{n_{X_i}(t_{\mathrm{in,\ tunn}})}{n_{\mathrm{tot}}(T_{\mathrm{in,\ tunn}})}\right) \quad (9)$$

is the sum of the branching ratios after tunneling from the initial throat for the X_i modes in the throats $i \neq \{\mathrm{SM}, \mathrm{in}\}$ weighted by the ratio of their masses m_{X_i} and the lightest KK mass $m_{KK(\mathrm{in})}$ in the initial throat[9].

In order to avoid overclosure of the Universe by the KK dark matter particles, the total relic density given by Eq. (8) has to be less than the 3σ upper limit of the WMAP observed value [20]: $\Omega_X h^2 \leq 0.13$. The allowed parameter space satisfying this condition is shown in Fig. 1. As the dependence on the total numbers of relativistic degrees of freedom at different temperatures is not strong, we have assumed the common value 250 at early times, to account for the pure MSSM sector plus some hidden degrees of freedom.

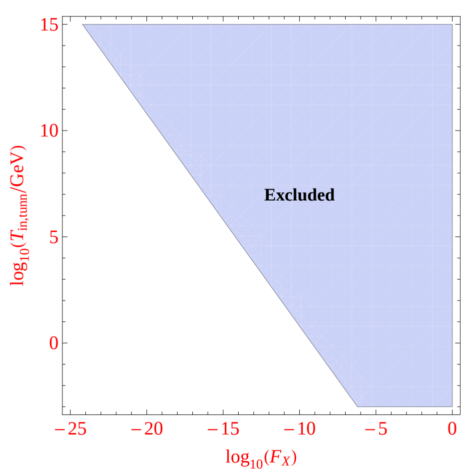

FIG. 1. The allowed parameter space satisfying the overclosure constraint. We have taken $g_{*S}(T_{\mathrm{in,\ tunn}}) \simeq g_*(T_{\mathrm{in,\ tunn}}) = 250$.

The condition of SM radiation domination after tunneling from the initial throat requires the SM throat to be the one with the densest spectrum, which favors tunneling as the matching between energy levels is easier. This implies

$$h_i R_i^{-1} > h_{\mathrm{SM}} R_{\mathrm{SM}}^{-1}. \quad (10)$$

[9] Note that in deriving Eq. (8) we do not make any assumption (relativistic/non-relativistic or even thermal/non-thermal) on the energy or number density of the modes X_i at $t_{\mathrm{in,\ tunn}}$. The final expression of the relic density in Eq. (8) is however similar to those in [21, 22].

The radiation domination further depends on the thermal history of the degrees of freedom in the SM throat. The modes tunneled there can 1) decay into the lowest KK(SM) modes, 2) thermalize, 3) decay into SM fields. In the allowed range of warping, the decay into the SM radiation [14, 23] is very rapid, i.e., less than one Hubble time $\sim H^{-1}_{\mathrm{in,\ tunn}} \sim \Gamma^{-1}_{\mathrm{in,\ tunn}}$ right after they have tunneled, provided that

$$\Gamma_{\mathrm{SM}} = \frac{m^3_{\mathrm{KK,\ SM}}}{M^2_{\mathrm{SM}}} = h_{\mathrm{SM}} \frac{\ell_S^2}{R^3_{\mathrm{SM}}} \gg \Gamma_{\mathrm{in,\ tunn}}. \quad (11)$$

Because of Eq. (10), this also entails a lower bound on the warping of the hidden throats.

It is possible that mildly broken isometries could exist at the tip of the initial throat [24]. In this case, the KK(in) modes possessing the related approximately conserved quantum number would be long-lived (LL) modes. They would not easily tunnel out because of difficult quantum number matching between throats, so that the other DM and SM throats will not be populated by these dangerous LL modes. Their present relic density reads

$$\Omega_{\mathrm{LL_{in}}} h^2 = \frac{3.24 \times 10^{26}}{g_s^2 \widetilde{g}_* \bar{g}_*} \frac{M^2_{\mathrm{in}}}{M^2_{\mathrm{PL}}} \frac{T_{\mathrm{in,tunn}}}{m_{X_{\mathrm{in}}}} \left(\frac{m_{\mathrm{LL_{in}}}}{T_{\mathrm{dec,LL_{in}}}}\right)^{\frac{3}{2}}, \quad (12)$$

where $T_{\mathrm{dec,LL_{in}}}$ is the decoupling temperature of the LL modes with mass $m_{\mathrm{LL_{in}}}$, given by

$$\frac{m_{\mathrm{LL_{in}}}}{T_{\mathrm{dec,LL_{in}}}} = \log\left(\frac{\sqrt{3}}{(2\pi)^{3/2}} g_*(T_{\mathrm{dec,LL_{in}}})^{\frac{1}{2}} g_s^2 \frac{M_{\mathrm{PL}} m_{\mathrm{LL_{in}}}}{M^2_{\mathrm{in}}}\right), \quad (13)$$

and M_{in} is the local string scale in the initial throat. We have also defined

$$\widetilde{g}_* \equiv \frac{g_{*S}(T_{\mathrm{dec,LL_{in}}}) g_*(T_{\mathrm{RDMD}})^{\frac{3}{4}}}{g_*(T_{\mathrm{dec,LL_{in}}})^{\frac{1}{2}} g_{*S}(T_{\mathrm{RDMD}})} \quad (14)$$

$$\bar{g}_* \equiv \frac{g_*(T_{\mathrm{in,MD}}) g_{*S}(T_{\mathrm{in,\ tunn}})}{g_{*S}(T_{\mathrm{in,MD}}) g_*(T_{\mathrm{in,tunn}})}, \quad (15)$$

where $T_{\mathrm{in,MD}}$ is the temperature when the lightest KK mode in the initial throat becomes non-relativistic, and T_{RDMD} is the temperature at the standard transition between radiation and matter domination. Eqs. (12) and (13) match the standard result for cold relics [25], with the additional $\bar{g}_*^{-1} \frac{T_{\mathrm{in,\ tunn}}}{m_{X_{\mathrm{in}}}}$ suppression due to the matter domination (MD) period between the time when the lightest KK modes in the initial throat become non-relativistic and the end of tunneling from the initial throat.

The relic density (12) for the LL(in) modes is plotted in Fig. 2 for some typical string coupling values. In this plot, we show the case where the throat geometry is well-approximated by the $AdS_5 \times X_5$ RS-like picture, and we have used $R_{\mathrm{in}} = 10\ell_s$ and $M_s \sim 10^{16}$

GeV (GUT scale). As before, due to the weak dependence on the total numbers of relativistic degrees of freedom at different temperatures, we have assumed the common value 250 at early times. We

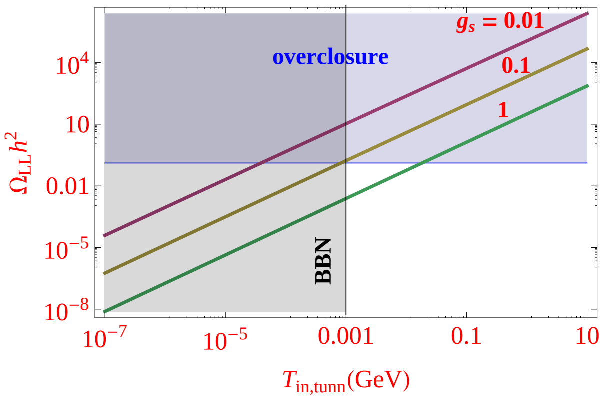

FIG. 2. The relic density of the long-lived KK modes in the initial throat with RS-like geometry as a function of the tunneling temperature. Here we have chosen $R_{\rm in} = 10\ell_s$ and $M_s = 10^{-2} M_{\rm PL}$. The vertical shaded region is excluded from BBN constraints. In the horizontal shaded region, these LL modes will overclose the Universe. Note that for the string coupling $g_s \leq 0.1$, these LL modes, if present, are always dangerous relics.

see that unless the interactions of these LL modes are strong enough (for $g_s \gtrsim 1$), their relic density will be too large in the allowed temperature range, i.e. $T_{\rm in,tunn}, T_{\rm dec,LL_{in}}$ larger than the Big Bang Nucleosynthesis (BBN) temperature ~ 1 MeV. Thus, we conclude that for reasonably small values of the string coupling $g_s \leq 0.1$, we must not have the long-lived KK modes in the initial throat. In other words, all the isometries must be broken (which usually happens due to supergravity effects, and we will assume in the rest of this subsection) in order for the Universe not to be overclosed. Increasing the value of $R_{\rm in}$ worsens the situation, and only a lower value of M_s would improve it. For the cases where the decay rate is more suppressed, as in the models of [15], the constraint is even less easy to satisfy (for the values of the parameters chosen for Fig. 2, even the case $g_s = 1$ is ruled out for the LL modes).

Beside overclosure, there are other cosmological constraints to be satisfied as well. First of all, the tunneling decay must occur well before the BBN in the SM throat which happens at temperature $T_{\rm BBN} \sim 1$ MeV, and sets the lower bound for $T_{\rm in,tunn}$. Furthermore, for our four-dimensional field theory description to be valid, we should have the reheating temperature in each throat lower than the local string scale in order to avoid exciting stringy degrees of freedom, which entails a lower bound on the local string scale (or, on the warp factor, and thus, on the KK mass) in each throat:

$$M_i \gtrsim T_{\rm in,\,tunn} \Rightarrow m_{X_i} \gtrsim \frac{\ell_s}{R_i} T_{\rm in,\,tunn}. \qquad (16)$$

It may be noted here that a more realistic lower bound on the warp factor might be obtained by considering the throat deformation due to moduli destabilization in a Hubble expansion background [26, 27]. However, this is very hard to compute explicitly, and hence, we use the more intuitive bound in Eq. (16).

The bound from BBN temperature, the condition Eq. (16), the relic density overclosure constraint, the requirements of rapid and efficient SM reheating (i.e., Eqs. (10)-(11)), and the stability condition in each throat, which reads

$$\Gamma_{{\rm tunn}(i)} < H_0 \simeq 1.5 \times 10^{-42}\ {\rm GeV}, \qquad (17)$$

put a strong constraint on the warping factors in the throats harboring the KK modes, which is directly related to the DM masses m_{X_i} and the local string scales M_i in the throats.

We have studied the allowed region of parameter space (warpings) after imposing these conditions. The relation between the warping $h_{\rm in}$ and $T_{\rm in,\,tunn}$ follows from the dependence of $\Gamma_{\rm in,\,tunn}$ on $h_{\rm in}$, which in turns depends on the form of the throat metric. We present in Fig. 3 the result for different fractions F_X for the case when the throat geometry is well approximated by a RS-like $AdS_5 \times X_5$ picture, and will discuss later the results for models with more suppressed tunnelings as those in [15]. In Fig. 3 the constraints are shown as various shaded regions and the allowed parameter space after imposing all the constraints is shown as the white (unshaded) region. We have used $R_i \simeq R_{\rm in} \simeq R = 10\ell_s$ and $M_s \sim 10^{16}$ GeV (GUT scale) [10]. With the above choices, the allowed mass range for the DM candidates is limited to about 0.1 MeV - 10 TeV, with the local string scale in the initial throat at most at $\sim 10^{12}$ GeV.

In the case of models where the tunneling rate is more suppressed by higher powers of the warping than in the case of RS-like geometry of the throats (see section II), the allowed region of parameter space shrinks, and the warping of the initial throat has to be shallower, i.e., $m_{X_{\rm in}}$ and $M_{\rm in}$ have to be larger. For the most suppressed case with $\Gamma \sim h^{17} R^{-1}$ for

[10] The assumtion that the curvature radius R is similar for all throats is justified by the fact that in string theory flux compactification typically $R \sim (F)^{\frac{1}{4}} g_s^{\frac{1}{4}} \ell_s$ [11, 19, 28] (this is certainly true when the throat metric is of the $AdS_5 \times X_5$ RS-like kind), where F is given by some flux/brane numbers which can vary from throat to throat. Even if F varies by a factor 10^6, R differs just by about one order of magnitude.

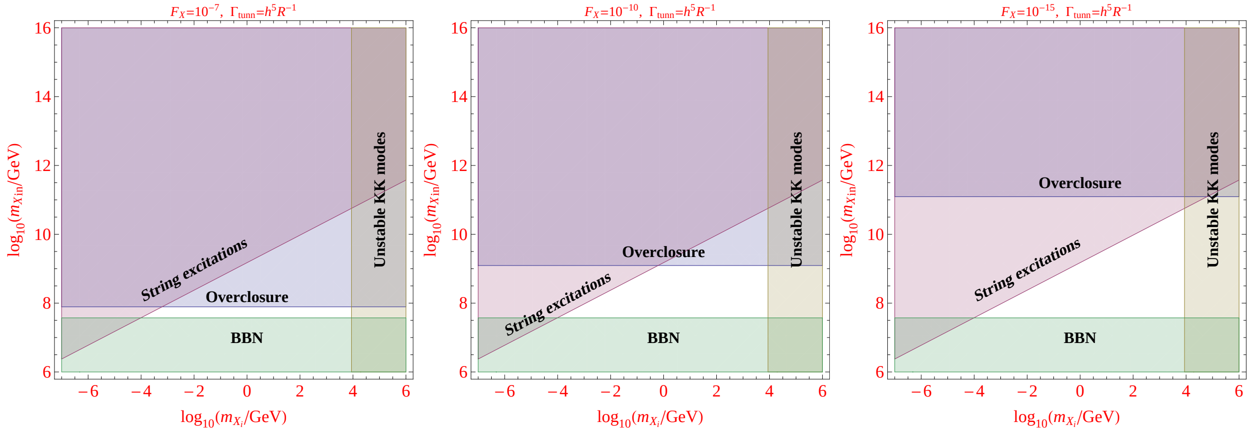

FIG. 3. The allowed range, shown as the white (unshaded) region, for different fractions F_X defined by Eq. (9) and arbitrary number of throats in terms of the lightest KK masses in the initial throat ($m_{X_{\rm in}}$) and other hidden throats (m_{X_i}). We plot here the case of $AdS_5 \times X_5$ RS-like throat geometry (for other cases see text). Here, we have chosen the string scale to be $M_{\rm PL}/100$ and the AdS radius of curvature in each throat to be $10M_{\rm PL}^{-1}$, where $M_{\rm PL} = 2.4 \times 10^{18}$ GeV is the reduced Planck mass. Note that the condition (10) in combination with (11) is automatically satisfied for this range of parameters and we have not shown it in the plot.

the throats, the mass scales are found to be quite restricted; for instance, $m_{X_{\rm in}}$ can only vary within about an order of magnitude.

We have also studied how the allowed region of parameter space varies with the parameters R and M_s. Keeping the latter fixed and letting R vary, the allowed region shrinks (both for RS-like throat geometry and for the cases with more suppressed decay rates) and moves to lower mass scales However, we find that the mass of the DM candidates cannot be lower than about 0.1 eV satisfying all the constraints along with the condition in Eq. (11). Also, keeping R fixed, the upper bounds on the KK mass scales in the initial and other hidden throats get lower when M_s gets smaller (without affecting much the lower bound on m_{X_i}), and the allowed region again shrinks.

Interestingly, when the throat geometry is well approximated by an $AdS_5 \times X_5$ RS-like picture, for all values of R and M_s, we find that the local string scale in the initial throat $M_{\rm in}$ has to be lower than about 10^{13} GeV in the compactified effective theory (i.e., with stabilized throat geometry). If inflation take places in this throat, then, its scale cannot be larger than the local string one, and this would rule out a high scale (around GUT-scale) multi-throat inflationary scenario in this case. This in turn puts a strong constraint on the tensor-to-scalar ratio, and hence, the possibility of detecting primordial gravitational waves (see e.g. [9]).

B. Reheating in the bulk

In this scenario the energy density responsible for reheating is not localized in a throat and couples without tunneling suppression to all throats. Although this initial energy density can be stored in bulk modes or moduli fields different than the inflaton, for simplicity we will call "inflatons" all the degrees of freedom associated with the initial energy density. We assume that the KK modes are produced directly by the inflaton decay at reheat temperature T_R within the throats where they are localized.

As discussed earlier, the KK modes in different long throats will be decoupled from each other because of tunneling suppression of the relevant interaction cross-sections. The present DM abundance can be estimated, assuming SM radiation domination after reheating, and without making assumptions regarding the energy/number density of the X_i modes at the reheating time t_R. We obtain a result similar to Eq. (8), with $T_{\rm in,tunn}$ replaced by T_R and $F_X = \frac{\sum_i m_{X_i} n_{X_i}(t_{\rm R})}{\rho_{\rm tot}(T_{\rm R})}$, where $\rho_{\rm tot}(T_{\rm R}) = 0.3 g_*(T_R) T_R^4$.

Differently than the throat-reheating scenario, in the bulk-reheating case we are discussing here, the SM fields can be reheated directly since the "inflaton" is directly coupled to them. In this case, we do not have to require the condition (11) for SM radiation to be produced rapidly. However, the inflaton is coupled to the other throats as well and with the bulk; hence, there could easily be overproduction of the non-SM degrees of freedom and/or gravitons. The efficiency of these processes are however model-dependent and

beyond the scope of our discussion here [11].

Another important difference with the throat-reheating case concerns the possibility of long-lived modes because of mildly broken isometries [24]. We recall that in the throat-reheating case all the energy density comes from the initial throat by tunneling, and LL modes in that throat will not easily tunnel out because of difficult quantum number matching. Thus, the DM and SM throat will not be populated by dangerous long-lived KK modes. But in the case of bulk reheating, dangerous LL modes can be excited in all throats, if mildly broken isometries exist, due to the direct coupling with the reheating (inflaton) sector. Their relic density in the SM throat has the same form as Eq. (12) *without* the suppression factor due to matter domination before tunneling, and substituting all "in"-labels with the label "SM" indicating the throat in question:

$$\Omega_{\mathrm{LL_{SM}}} h^2 = \frac{3.24\times 10^{26}}{g_s^2 \widetilde{g}_*} \frac{M_{\mathrm{SM}}^2}{M_{\mathrm{PL}}^2} \left(\frac{m_{\mathrm{LL_{SM}}}}{T_{\mathrm{dec,LL_{SM}}}}\right)^{\frac{3}{2}}, \qquad (18)$$

where the decoupling temperature is obtained by similarly substituting the "in"-labels with "SM" in Eq. (13). From Eq. (18), we can see that in the presence of LL modes in the SM throat, the overclosure bound will constrain the warping factor h_{SM}. For instance, for $M_s \sim 10^{16}$ GeV, $g_s \sim 0.1$ and $R_{\mathrm{SM}} = 10\ell_s$, we get $h_{\mathrm{SM}} \lesssim 10^{-14}$. The bound gets stronger if the interactions of the LL modes are weaker (i.e., g_s is smaller or M_s is increased). For each of the other hidden throats, the LL(j) relic density will be determined by the effective decoupling among throats due to tunneling suppression.

We have therefore two scenarios for the requirement that stable KK modes do not overclose the Universe in the bulk reheating case, depending on if the mildly broken isometries responsible for the presence of long-lived modes exist or not. In the latter case, the total relic density is just $\Omega_{\mathrm{X}} h^2$ given by Eq. (8) with $t_{\mathrm{in,tunn}}/T_{\mathrm{in,tunn}}$ replaced by t_R/T_R. But if there are some isometries and the relative LL modes, the total relic density is obtained by including in F_{X} the number density and mass of the LL(j)-modes.

As before, one might ask here for the reheating temperature to be lower than the string scale in the throats, to avoid exciting stringy degrees of freedom, which entails $M_i \gtrsim T_R$. This, together with the relic density overclosure constraint and the requirement of stability of the KK modes, leads to bounds on the mass scale of the KK modes. We first discuss the results for the case when the throat geometry is well approximated by an $AdS_5 \times X_5$ RS-like picture. This is shown in Fig. 4 for the choice $R_i \simeq R_{\mathrm{in}} = 10\ell_s$ and $M_s \sim 10^{16}$ GeV. Note that in the bulk-reheating case, if there are long-lived KK modes in the throats (as shown by the dashed-region in Fig. 4), the allowed mass range for the KK modes is even smaller, i.e. between 0.1 MeV-100 GeV. Varying R_i and M_s, or considering the non-RS-like geometries such as in [15], leads to the same variation of scales and shrinking of the allowed region discussed in the previous subsection. However in this case, it is not possible to obtain in a model-independent manner a minimum absolute lower bound on m_{X_i} or a relation between the maximum T_R and the scale of inflation.

[11] For instance, in large volume compactification models, it is well-known that the inflaton branching ratio to the hidden degrees are much more than the visible sector degrees of freedom [29].

IV. SOME PHENOMENOLOGICAL IMPLICATIONS

In this section, we briefly discuss some interesting phenomenological consequences of the multiple DM scenario advocated here. For other aspects of the multiple DM phenomenology, see e.g. [16, 17, 30].

(i) The total DM relic density will be given by the sum of the relic densities in the visible and hidden sectors:

$$\Omega_{\mathrm{DM}}^{\mathrm{total}} h^2 = \Omega_{\mathrm{DM}}^{\mathrm{visible}} h^2 + \Omega_{\mathrm{DM}}^{\mathrm{hidden}} h^2\,, \qquad (19)$$

where $\Omega_{\mathrm{DM}}^{\mathrm{hidden}} h^2$ includes the contribution of the KK dark matter particles and, possibly, other stable and weakly interacting matter from other hidden sectors. This implies that the lower limit of the WMAP measured value: $\Omega_{\mathrm{CDM}} h^2 > 0.09$ [20] is no more applicable to the visible sector DM candidate. This releases a large part of the parameter space in many visible sector DM models. For example, this provides one way to salvage the constrained MSSM with neutralino LSP in the light of the recent LHC data, even though the additional parameter space available for $\Omega h^2 < 0.09$ [31] is not spectacularly large.

(ii) The KK dark matter candidates in our multi-throat scenario are allowed to be light, and hence, could possibly serve as warm (or even hot) dark matter candidates (if their velocity distribution is the opportune one). If the KK modes are sufficiently light (of order eV), they could act as extra sterile degrees of freedom (similarly to sterile neutrinos [32]) from the point of view of the visible sector, thus possibly explaining the deviation of the measured value of the effective number of relativistic degrees of freedom at BBN, $N_{\mathrm{eff}}^{\mathrm{WMAP}} = 4.34 \pm 0.87$ [20], from the SM value $N_{\mathrm{eff}}^{\mathrm{SM}} = 3$. One of the main constraints on the number of sterile degrees of freedom is provided by the high sensitivity of the expansion rate of the Universe

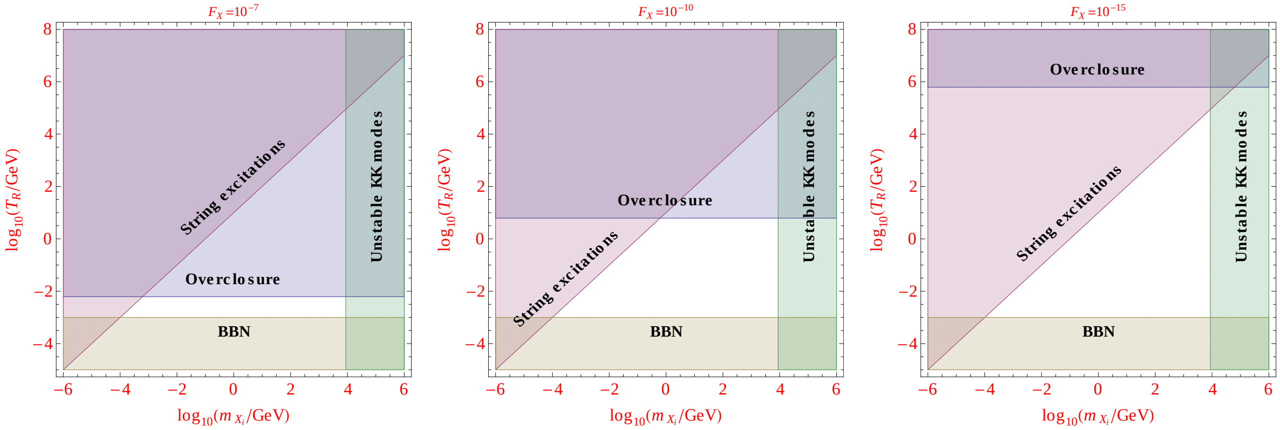

FIG. 4. The allowed range, shown as the white (unshaded) region, of the reheating temperature (T_R) and the lightest KK mass in the hidden throats (m_{X_i}) in the bulk reheating case for different energy density fractions F_X and arbitrary number of throats. We plot here the case of $AdS_5 \times X_5$ RS-like throat geometry (for other cases see text). Here we have chosen the string scale to be $M_{\rm PL}/100$ and the radius of curvature in each throat to be $10M_{\rm PL}^{-1}$, where $M_{\rm PL} = 2.4 \times 10^{18}$ GeV is the reduced Planck mass.

at the BBN temperature: $T_{\rm SM,BBN} \sim 1$ MeV. Sufficiently light KK modes could change the expansion rate of the Universe and thereby impact the BBN, even though they do not reside in the SM throat, and do not have any SM interactions. This can be translated into a bound (see e.g. [2])

$$g_{\rm KK}\left(\frac{T_{\rm KK,BBN}}{T_{\rm SM,BBN}}\right)^4 = \frac{7}{8}\cdot 2\cdot (N_{\rm eff}-3) \qquad (20)$$

where $g_{\rm KK}$ is the number of internal degrees of freedom for the relativistic KK modes. This bound could be satisfied in our scenario depending on the precise value of the KK dark matter temperature in the hidden throats.

(iii) In the scenario where inflation occurs in the initial throat, the bounds on the local mass scale also constrain the scale of inflation in the compactified effective theory (for which we must have $R_i > \ell_S$, $M_s < M_{\rm PL}$). In particular, when the throat geometry is well approximated by the $AdS_5 \times X_5$ RS-like picture, the local string scale, and hence the scale of inflation, in the initial throat is found to be $\lesssim 10^{13}$ GeV. This would put a strong constraint on building any inflationary model with large tensor-to-scalar ratio, which would generate detectable gravitational waves during or after inflation (see e.g. [9]). Therefore, if such a large tensor-to-scalar ratio is observed (e.g., by the Planck satellite), this would certainly disfavor the multi-throat warped models with RS-like throat geometry where inflation occurs in a hidden throat, since the Universe will be inevitably overclosed by the KK relics.

(iv) Depending on the model construction, the KK modes residing in the SM throat could decay to light supergravity modes and/or directly to the SM particles, thus leading to some interesting indirect detection signals [7, 33]. Very light modes in the SM throat could also provide observable effects for example in the gravity sector, and hence, a test for our model.

(v) Given a number C of conifold points in the Calabi-Yau manifold and the upper bound $h_{\rm max}$ on the warping factor derived in the previous section, we can use Eq. (4) to obtain an estimate of the average number of throats long enough to harbor the stable KK modes:

$$\bar{N}(h < h_{\rm max}|C) \sim 0.01C. \qquad (21)$$

Thus, roughly 1% of the total number of conifold points can be possible dark matter candidates in our multi-throat scenario.

V. CONCLUSION

We have discussed the possibility of having multiple dark matter candidates in the form of the lightest KK-modes residing in various throats which arise naturally out of generic Calabi-Yau compactification in string theories. The stability of these KK-modes is due to the small tunneling rate because of the background warped geometry. Compliance with cosmological and consistency constraints imposes strong bounds on the underlying string compactified models. However, we have shown that there exists some parameter space where the KK modes can be DM candidates. This gives us a very different picture for particle dark matter than the usually discussed *single* WIMP scenario. We also note here that relaxing the stability constraint for the DM states and instead requiring only a dynamical balance against their abundance, as in the

so-called Dynamical DM scenario [17], might open up a larger parameter space in our case.

As a word of caution, we must keep in mind the generality of the models we have considered, by neglecting the precise couplings between the reheating and the reheated degrees of freedoms in the bulk-reheating scenario, which preclude us from knowing the exact branching ratios, and assuming pure KK matter content in the hidden throats and not other possible more complicated matter sectors as well as radiation. We leave some of these issues for future investigation.

ACKNOWLEDGMENTS

The work of DC is supported by the Belgian National "Fond de la Recherche Scientifique" F.R.S.-F.N.R.S. with a contract "chargé de recherche". The work of BD and AM is supported by the Lancaster-Manchester-Sheffield Consortium for Fundamental Physics under STFC grant ST/J000418/1.

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